Application of partial Differential Eqⁿ.

Method of separation of variable

Q Solve (by the method of ~~variation~~ separation of variable)

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0.$$

$z = f(x, y)$

$\frac{\partial z}{\partial x}$ - y-cons

$\frac{\partial z}{\partial y}$ - x cons.

Solⁿ. Given differential equation is

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0 \quad \text{——} (1)$$

Let

$z = X_{(x)} Y_{(y)}$ be a solution of equation (1)

$\frac{\partial z}{\partial x} = X'Y$ and $\frac{\partial z}{\partial y} = XY'$

and $\frac{\partial^2 z}{\partial x^2} = X''Y$.

Substituting these values in equation (1), we get

$$X''.Y - 2X'Y + XY' = 0.$$

$$\Rightarrow (X'' - 2X')Y = -XY'$$

$$\Rightarrow \frac{X'' - 2X'}{X} = \frac{-Y'}{Y} = K \text{ (say)}$$

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$\Rightarrow \dfrac{X'' - 2X'}{X} = K$

$\Rightarrow X'' - 2X' - XK = 0$

SF

$\Rightarrow [D^2 - 2D - K]X = 0$

AE

$D^2 - 2D - K = 0$

$D = \dfrac{2 \pm \sqrt{4+4K}}{2}$

$= 1 \pm \sqrt{1+K}$

$X = C_1 e^{(1-\sqrt{1+K})x} + C_2 e^{(1+\sqrt{1+K})x}$

$\dfrac{-Y'}{Y} = K$

$\Rightarrow -Y' - YK = 0$

or

$\Rightarrow Y' + YK = 0$

SF

$(D_1 + K)Y = 0$

AE

$(D_1 + K) = 0$

$D_1 = -K$

$Y = C_3 e^{-ky}$

Solution of given differential equation is

$Z = XY$

$\Rightarrow Z = [C_1 e^{(1-\sqrt{1+K})x} + C_2 e^{(1+\sqrt{1+K})x}][C_3 e^{-ky}]$

$\Rightarrow Z = [A e^{(1-\sqrt{1+K})x} + B e^{(\sqrt{1+K}+1)x}] e^{-ky}$

Problem 2 Using the method of separation of variable solve, $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ —— (1) where $U(x,0) = 6e^{-3x}$ —— (2)

Let $U = X_{(x)} T_{(t)}$

$\frac{\partial u}{\partial x} = X'T$, $\frac{\partial u}{\partial T} = XT'$

$X'T = 2XT' + XT$

$\Rightarrow X'T - XT = 2XT'$

$\Rightarrow \frac{X' - X}{2X} = \frac{T'}{T} = K$

$\Rightarrow X' - X = 2KX$ | $T' = TK$

$\Rightarrow X' - X - 2KX = 0$ | $\Rightarrow T' - TK = 0$

$\Rightarrow X' - X(1+2K) = 0$ | $\Rightarrow T' - TK = 0$

SF $[D - (1+2K)]X = 0$ | SF $\Rightarrow [D_1 - K]T = 0$

AE $\Rightarrow D - (1+2K) = 0$ | AE $\Rightarrow D_1 - K = 0$

$\Rightarrow D = 1 + 2K$ | $\Rightarrow D_1 = K$

$X = C_1 e^{(1+2K)x}$ | $T = C_2 e^{Kt}$

$U = XT$

$U = C_1 e^{(1+2K)x} \times C_2 e^{Kt} = A e^{(1+2K)x + Kt}$ —— (3)

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Now,

$$U(x,0) = 6e^{-3x} \qquad \text{(Given)}$$

$$A e^{(1+2k)x + k(0)} = 6e^{-3x}$$

$$\Rightarrow \quad A e^{(1+2k)x} = 6e^{-3x}$$

Equating like terms,

$$A = 6, \qquad e^{(1+2k)x} = e^{-3x}$$

or

$$(1+2k)\cancel{x} = -3\cancel{x}$$

$$\Rightarrow \quad 1+2k = -3$$

$$\Rightarrow \quad \boxed{k = -2}$$

$$\therefore \quad U(x,t) = 6e^{[1-2\times 2]x + (-2)t}$$

$$\Rightarrow \boxed{U(x,t) = 6e^{-3x-2t}} \quad \text{Ans}$$

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Problem 3 Solve $\frac{4\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$,

given that $u = 3e^{-y} - e^{-5y}$ when $x = 0$.

Soln The given equation is

$$\frac{4\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u \quad \text{--- (1)}.$$

let $u = X_{(x)} Y_{(y)}$ be a solution of eqn (1)

$$\frac{\partial u}{\partial x} = X'Y \qquad \frac{\partial u}{\partial y} = XY'$$

Substitute these values in Eq (1)

$$4X'Y + XY' = 3XY$$

$$\Rightarrow (4X' - 3X)Y = -XY'$$

let

$$\Rightarrow \frac{4X' - 3X}{X} = \frac{Y'}{Y} = K$$

$$\Rightarrow \frac{4X' - 3X}{X} = K \qquad\qquad -\frac{Y'}{Y} = K$$

$$\Rightarrow 4X' - 3X - KX = 0 \qquad\qquad \Rightarrow -Y' - KY = 0.$$

$$\Rightarrow 4X' - (K+3)X = 0 \qquad\qquad \text{SF} \Rightarrow [D + K]Y = 0$$

SF

$$\Rightarrow [4D - (K+3)]X = 0 \qquad\qquad \Rightarrow \text{AE} \quad D + K = 0$$

AE

$$4D - (K+3) = 0 \qquad\qquad \Rightarrow \underline{D = -K}$$

$$D = \frac{K+3}{4}$$

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$X = c_1 e^{\left(\frac{k+3}{4}\right)x}$ | $Y = c_2 e^{ky}$

$U = XY$

$\Rightarrow U = c_1 e^{\left(\frac{k+3}{4}\right)x} \cdot c_2 e^{ky}$

$\Rightarrow U = A e^{\left(\frac{k+3}{4}\right)x + ky}$

Given, ~~U(0,y)~~ $U(0,y) = 3e^{-y} - e^{-5y} = Ae^{-ky}$

Now, Ae^{-ky} can be either equal to $3e^{-y}$ or $-e^{-5y}$

$Ae^{-ky} = 3e^{-y}$

$\Rightarrow A = 3$ and $k = 1$

$U(x,y) = 3e^{x-y}$

$Ae^{-ky} = -e^{-5y}$

$A = -1$ and $k = +5$

$U(x,y) = -e^{2x-5y}$

Required Sol^n is

$U(x,y) = 3e^{x-y} - e^{2x-5y}$

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Q Solve the wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \quad \text{—— (1)}$$

Soln: let $y = X_{(x)} T_{(t)}$ be the solution of equation (1)

$$\frac{\partial y}{\partial x} = X'T \quad \text{and} \quad \frac{\partial y}{\partial t} = XT'$$

$$\frac{\partial^2 y}{\partial x^2} = X''T \quad \text{and} \quad \frac{\partial^2 y}{\partial t^2} = XT''$$

Substitute these values in equation (1),

$$XT'' = c^2 X'' T$$

$$\Rightarrow \frac{X''}{X} = \frac{T''}{c^2 T} = k \text{ (say)}$$

$$\frac{X''}{X} = k$$

$$\Rightarrow X'' - kX = 0$$

SF

$$[D^2 - k]X = 0$$

AE

$$D^2 - k = 0$$

$$\frac{T''}{c^2 T} = k$$

$$\Rightarrow T'' - c^2 T k = 0$$

SF

$$[D^2 - c^2 k]T = 0$$

AE

$$D^2 - c^2 k = 0$$

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Case I If $k<0$, let $k=-p^2$ $p\neq 0$.

$D^2-k=0$

$\Rightarrow D^2+p^2=0$

$\Rightarrow D=\pm pi$

$X = c_1\cos px + c_2\sin px$

$D_1^2-c^2(k)=0$

$D_1^2-c^2(-p^2)=0$

$D_1^2+p^2c^2=0$

$\Rightarrow D_1=\pm pci$

$T= c_3\cos pct + c_4\sin pct$ —— (2)

$y = XT$

$y(x,t) = [c_1\cos px + c_2\sin px][c_3\cos cpt + c_4\sin cpt]$

Case II If $k>0$, let $k=p^2$ $p\neq 0$

$D^2-k=0$

$D^2-p^2=0$

$D=\pm p$

~~$X = c_5\cos px + c_6\sin px$~~

$X = c_1e^{px} + c_2e^{-px}$

$D_1^2-c^2k=0$

$D_1^2-p^2c^2=0$

$D_1=\pm pc$

$c_7e^{pc} + c_8e^{-pc}$

$y(x,t) = [c_1e^{px}+c_2e^{-px}][c_7e^{pct}+c_8e^{-pct}]$ —— (3)

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Case III If $k = 0$

$$D^2 - k = 0$$
$$\Rightarrow D^2 = 0$$
$$\Rightarrow D = 0, 0$$

$$X = (C_9 + x C_{10})$$

$$D^2 - c^2 k = 0$$
$$\Rightarrow D^2 = 0$$
$$\Rightarrow D = 0, 0$$

$$T = (C_{11} + t C_{12})$$

$$\boxed{y(x,t) = (C_9 + x C_{10})(C_{11} + t C_{12}).} \quad \text{— (4)}$$

$$y(x,t) = [C_1 \cos px + C_2 \sin px][C_3 \cos cpt + C_4 \sin cpt] \quad \text{— (2)}$$
$$y(x,t) = [C_5 e^{-px} + C_6 e^{px}][C_7 e^{-cpt} + C_8 e^{cpt}] \quad \text{— (3)}$$
$$y(x,t) = [C_9 + x C_{10}][C_{11} + t C_{12}] \quad \text{— (4)}$$

Out of these, we have to choose that solution which is consistent with the physical nature of the problems

As we are dealing with problems on vibrations (waves) y must be a periodic function of x and t. Hence their solution must involve trigonometric terms. Hence out of these solution, the possible solution is

$$y(x,t) = [C_1 \cos px + C_2 \sin px][C_3 \cos cpt + C_4 \sin cpt] \quad \text{Ans}$$

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II Solve the one dimensional heat flow equation.

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{——— (1)}$$

Solⁿ:- Let $U = X_{(x)} T_{(x)}$ be the solution of equation (1)

$$\therefore \quad \frac{\partial u}{\partial x} = X' T \qquad \frac{\partial u}{\partial t} = X T'$$

$$\frac{\partial^2 u}{\partial x^2} = X'' T \qquad \frac{\partial^2 u}{\partial t^2} = X T''$$

Substitute these values in equation (1).

$$\cancel{X'T} \quad XT' = c^2 X'' T$$

$$\Rightarrow \quad \frac{T'}{c^2 T} = \frac{X''}{X} = K \text{ (say)}$$

$\frac{X''}{X} = K$	$\frac{T'}{c^2 T} = K$
$\Rightarrow X'' - KX = 0$	$\Rightarrow T' - Kc^2 T = 0$
SF $\Rightarrow [D^2 - K]X = 0$	$\Rightarrow [D_1 - Kc^2]T = 0$ SF
AE $D^2 - K = 0$	$\Rightarrow D_1 - Kc^2 = 0$. AE

Case I $\quad K<0 \quad$ let $K=-p^2 \quad p\neq 0$

$$D^2-K=0$$
$$D^2+p^2=0$$
$$D=\pm pi.$$

$$\therefore X = C_1\cos px + C_2\sin px$$

$$D'-c^2K=0$$
$$D+c^2p^2=0$$
$$D=-c^2p^2$$
$$T=C_3e^{-c^2p^2t}$$

$$U(x,t) = XT$$
$$= [C_1\cos px + C_2\sin px]C_3e^{-c^2p^2t} \quad \text{—— (2)}$$

Case II $\quad K=0.$

$$D^2-0=0$$
$$\Rightarrow D=0,0.$$
$$X=(C_4+xC_5)$$

$$D-c^2(0)=0$$
$$\Rightarrow D=0$$
$$\Rightarrow T=C_6e^{0t}=C_6.$$

$$U(x,t)=XT$$
$$=[C_4+xC_5][C_6]. \quad \text{—— (3)}$$

Case III $\quad K>0 \quad K=p^2 \quad p\neq 0$

$$D^2-K=0$$
$$D^2-p^2=0.$$
$$D=\pm p.$$
$$\therefore X=[C_7e^{px}+C_8e^{-px}]$$

$$D-c^2K=0.$$
$$D-c^2p^2=0$$
$$D=c^2p^2$$
$$T=C_9e^{c^2p^2t}$$

$$U(x,t)=XT=[C_7e^{px}+C_8e^{-px}]C_9e^{c^2p^2t} \quad \text{—— (4)}$$

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(I) $U(x,t) = [c_1 \cos px + c_2 \sin px]\, c_3 e^{-c^2p^2t}$ ——— (2)

(II) $U(x,t) = [c_4 + x c_5][c_6]$ ——— (3)

(III) $U(x,t) = [c_7 e^{px} + c_8 e^{-px}]\, c_9 e^{c^2p^2t}$ ——— (4)

Out of these solution, we have to choose that solution which is consistent with the physical nature of the problem. As we are dealing with problems on heat condition, it must be transient solution i.e., u is to decrease with the increase of time t.

The suitable solution is given by eq.

$$\boxed{U(x,t) = [c_1 \cos px + c_2 \sin px]\, c_3 e^{-c^2p^2t}}$$ Ans

Q Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ — (1) [Laplace Equation]

Solⁿ

$u = XY$

$\frac{\partial^2 u}{\partial x^2} = X''Y \qquad \frac{\partial^2 u}{\partial y^2} = XY''$

$X''Y + XY'' = 0.$

$X''Y = -XY'' \Rightarrow \frac{X''}{X} = -\frac{Y''}{Y} = K \text{ (say)}$

$X'' = XK$

$\Rightarrow X'' - XK = 0$

SF $\Rightarrow (D^2 - K)X = 0$

$\Rightarrow$ AE $D^2 - K = 0$

$Y'' + KY = 0.$

$\Rightarrow Y'' + KY = 0$

SF $\Rightarrow [D_1^2 + K]Y = 0$

AE $D_1^2 + K = 0.$

Case I If $K > 0$, $K = p^2$, $p \neq 0$.

$D^2 - p^2 = 0.$

$\Rightarrow D^2 = p^2$

$\Rightarrow D = \pm p$

$X = [C_1 e^{px} + C_2 e^{-px}]$

$D_1^2 + K = 0.$

$D_1^2 + p^2 = 0$

$\Rightarrow D_1^2 = -p^2$

$\Rightarrow D_1 = -pi$

$Y = [C_3 \cos py + C_4 \sin py]$

$U(x, y) = [C_1 e^{px} + C_2 e^{-px}][C_4 \sin py + C_3 \cos py]$ — (11)

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<u>Case II</u> If $K<0$, $K=-p^2$, $p\neq 0$

$$D^2+p^2=0.$$
$$D=\pm pi$$
$$X=[C_5\cos px+C_6\sin px]$$

$$D_1^2 - p^2 = 0.$$
$$D_1^2 = p^2$$
$$D_1 = \pm p.$$
$$Y = C_7e^{py}+C_8e^{-py}$$

$$\therefore\ U(x,y) = [C_5\cos px + C_6\sin px][C_7e^{py}+C_8e^{-py}]$$

<u>Case III</u> If $K=0$, then

$$D^2+0=0$$
$$D=0,0.$$
$$X=[C_9+xC_{10}]$$

$$D_1^2-0=0.$$
$$D_1=0,0.$$
$$Y=[C_{11}+yC_{12}]$$

$$\therefore\ U(x,y)=[C_9+xC_{10}][C_{11}+yC_{12}]$$

Of these we taken that solution which is consistent with the given boundary condition.

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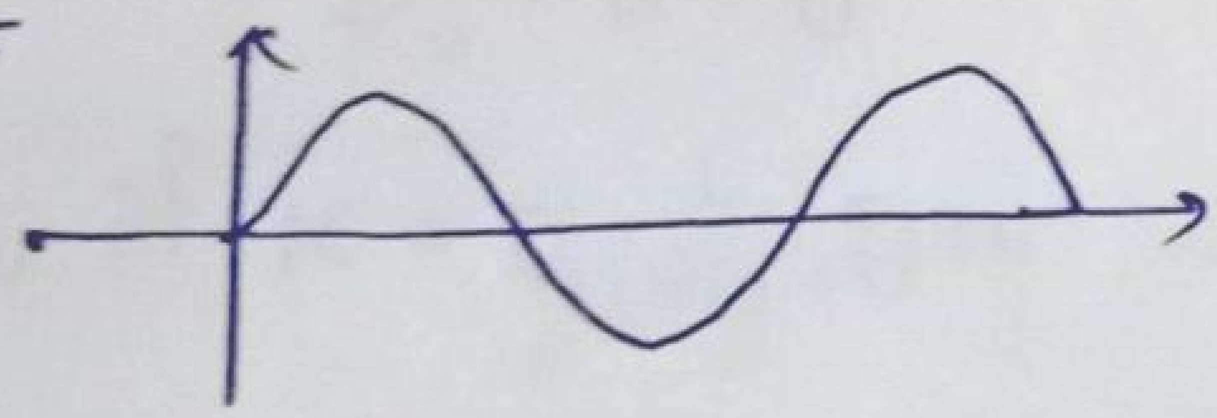
Problem: A string is stretched and fastened to two points l apart. Motion is started by displaying the string in the form $y = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time 't' $= 0$. Show that the displacement of any point at a distance x from one end at time t is given by

$$y(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi c t}{l}\right).$$

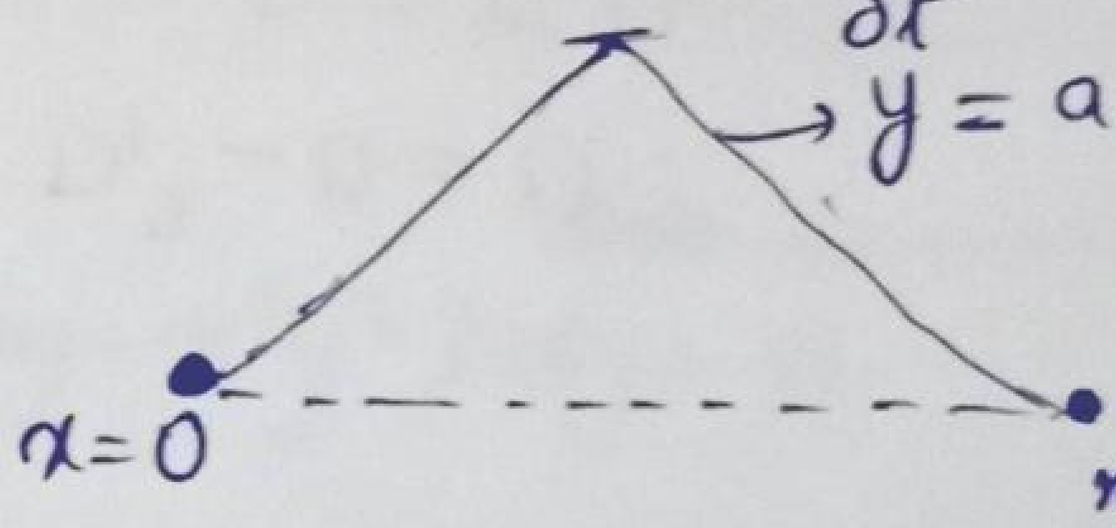
[y is independent of t]

Soln:-



$\frac{\partial y}{\partial t} = 0$

$y = a\sin\left(\frac{\pi x}{l}\right)$



The given problem is of wave motion and wave equation is given by

$$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2} \text{ ——— } (1).$$

Boundary condition are

$$y(0,t) = 0 \text{ ——— } (2)$$

$$y = a\sin\frac{\pi x}{l}$$

$$y(l,t) = 0 \text{ ——— } (3).$$

Initial condition

$$\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0 \text{ ——— } (4)$$

$$y(x,0) = a\sin\left(\frac{\pi x}{l}\right) \text{ ——— } (5).$$

The possible solution of equation is

$$y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos cpt + c_4 \sin cpt) \quad \text{--- (6)}$$

using (5) and (6)

$$y(0,t) = (c_1 + 0)(c_3 \cos cpt + c_4 \sin cpt)$$

$$0 = c_1 \underbrace{(c_3 \cos cpt + c_4 \sin cpt)}$$

$\Rightarrow c_1 = 0$ Otherwise, we willget zero solution

Substituting $c_1 = 0$ in (6), we get

$$y(x,t) = (0 + c_2 \sin px)[c_3 \cos cpt + c_4 \sin cpt] \quad \text{--- (7)}$$

using (3) and (4), $y(l,t) = 0$.

$$\cancel{y(l,t) = c_2 \sin pl\,[c_3 + 0]}$$

$$\cancel{0 = c_2 \sin pl \times c_3}$$

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$$y(l,t) = c_2 \sin pl \,(c_3 \cos cpt + c_4 \sin cpt)$$

$$0 = c_2 \sin pl \,\underbrace{[c_3 \cos cpt + c_4 \sin cpt]}_{\neq 0}$$

$c_2 \neq 0$. and Otherwise we will get zero solution

$\Rightarrow \sin pl = 0 = \sin n\pi$

$\Rightarrow n\pi = pl \Rightarrow \boxed{p = \frac{n\pi}{l}}$

Substitute this value in (4), we get

$$\rightarrow y(x,t) = c_2 \sin \frac{n\pi x}{l} \left[c_3 \cos \frac{n\pi c}{l} t + c_4 \sin \frac{n\pi c}{l} t\right] \quad \text{---} (8)$$

Diff. (8) partially w.r.t 't', we get

$$\frac{\partial y}{\partial t} = c_2 \sin \frac{n\pi x}{l} \left[-c_3 \frac{n\pi c}{l} \sin \frac{n\pi c}{l} t + c_4 \frac{n\pi c}{l} \cos \frac{n\pi c}{l} t\right] \quad \text{---} (9)$$

using (4) and (9), we get

$$\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0.$$

$$0 = c_2 \sin \frac{n\pi x}{l} \left[0 + c_4 \frac{n\pi c}{l}\right]$$

$\Rightarrow c_4 = 0$ [otherwise, we will get zero solution

$\therefore$ (8) reduces to

$$y(x,t) = c_2 \sin \frac{n\pi x}{l} * c_3 \cos \frac{n\pi c t}{l}$$

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$$\Rightarrow y(x,t) = b_n \sin\frac{n\pi x}{l} \cos\frac{n\pi c t}{l}.$$

G.S.

$$y(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \cos\frac{n\pi c t}{l} \quad \text{——} (10)$$

Using (5) in $y(x,0) = a\sin\frac{\pi x}{l}$ in (10) we get

$$a\sin\frac{\pi x}{l} = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$= b_1 \sin\frac{\pi x}{l} + b_2 \sin\frac{\pi 2}{l} + \cdots\cdots$$

on comparing,

we get ~~$a\sin\frac{\pi x}{l} = b_1 \sin$~~

$$b_1 = a \text{ and } b_2 = b_3 = 0 \ldots = b_n$$

Required solⁿ is

$$y(x,t) = \boxed{a\sin\frac{\pi x}{l} \cos\frac{\pi c t}{l}.} \text{ Ans}$$

Q. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary condition $U(x,0) = 3\sin n\pi x$, $U(0,t) = 0$, and $U(1,t) = 0$ where $0 < x < 1$, $t < 0$.

Solⁿ:- The given equation is $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ —— (1)

The possible solution is

$$\rightarrow U(x,t) = (c_1 \cos px + c_2 \sin px)\, e^{-p^2 t} \quad \text{—— (2)}$$

Now the given conditions are

$$U(0,t) = 0 \quad \text{—— (3)}$$
$$U(l,t) = 0 \quad \text{—— (4)}$$

$$U(x,0) = \cancel{c_1 \cos px} + 3_a \sin px \quad \text{—— (5)}$$

$0 < x < l$, $t > 0$

using (3) and (2) we get

$$U(0,t) = [c_1 + 0]e^{-p^2 t} = 0$$

$$\Rightarrow c_1 e^{-p^2 t} = 0 \qquad e^{-p^2 t} \neq 0.$$

$$\Rightarrow \boxed{c_1 = 0}$$

(2) reduces to

$$U(x,t) = [c_2 \sin px]e^{-p^2 t}$$

Using Eq (4) and (6), we get

$$U(l,t) = (c_2 \sin pl)e^{-p^2 t}$$

$$0 = c_2 \sin pl\, e^{-p^2 t}$$

$$\sin pl = 0 \Rightarrow \cancel{pl} = n\pi \Rightarrow p = \frac{n\pi}{\cancel{l}}$$

{ otherwise we will get zero solution

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$\therefore$ (6) reduces to

$$U(x,t) = C_2 \sin n\pi x \, e^{-n^2\pi^2 t} \quad \text{— (7)}.$$

General solution is given by

$$U(x,t) = \sum_{n=1}^{\infty} b_n \sin n\pi x \, e^{-n^2\pi^2 t} \quad \text{— (8)}$$

where $C_2 = b_n$.

using when $t=0$ $U(x,0) = 3\sin n\pi x$

$$3\sin n\pi x = \sum_{n=1}^{\infty} b_n \sin n\pi x$$

$$\Rightarrow \quad 3\sin n\pi x = b_1 \cancel{\sin \pi x} + b_2 \cancel{\sin 2\pi x} + \cdots\cdots b_n \sin n\pi x$$

$\Rightarrow$ $\boxed{3 = b_n}$.

$$\boxed{U(x,t) = \sum_{n=1}^{\infty} 3\sin \pi n x \, e^{-n^2\pi^2 t}.}$$

Ans.

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$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u = 0.$$

U is depending on x, y and x, y are independent

Second order partial differential equation of $u(x,y)$ written in the general form.

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial y^2} + C\frac{\partial^2 u}{\partial x \partial y} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu + G = 0.$$

If $B^2 - 4A$
- >0 : Hyperbolic
- $=0$: Parabolic
- <0 : Elliptic

Question

$$\frac{\partial^2 u}{\partial x^2} + 5\frac{\partial u}{\partial x} + 6u = 0.$$

Soln.

$$1 \cdot \frac{\partial^2 u}{\partial x^2} + 0 \cdot \frac{\partial^2 u}{\partial y^2} + 0 \cdot \frac{\partial^2 u}{\partial x \partial y} + 5\frac{\partial u}{\partial x} + 6u = 0$$

$A = 1$
$B = 0$
$C = 0$

$B^2 - 4AC = (0)^2 - 4(1)(0) = 0$

Parabolic